Dynamical and spectral defects in approximations of substitution systems

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The study of Schrödinger operators arising from crystals is a rich and well developed theory. When turning higher dimensional Schrödinger operators of quasicrystals, one often looks for suitable periodic approximations. Recent results by [BBDN18], [BBC19] and [BT25], show that when the underlying dynamical systems converge, the associated spectra converge to the limiting spectrum in the Hausdorff distance.
In a recent work [BBPT24] with Ram Band, Siegfried Beckus and Felix Pogorzelski we described a scheme to approximate aperiodic substitution subshifts. In [BBPT24] we gave algorithmic criteria for convergence and rate of convergence. In a follow-up work in progress, we elaborate on cases when the scheme fails to yield a successful approximation. We elaborate on the partial limits of the failed approximations and the defect which arise. In particular, we show that natural approximation candidates for the table tiling substitution yield worm-like impurities and spectral pollution in the essential spectrum. Moreover, we show that certain potential functions, yield spectral pollution of positive measure.
This talk is based on joint work with Ram Band, Siegfried Beckus and Felix Pogorzelski.

References

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