Novel Patterns in an Class of Klein-Gordon Equations

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Abstract

We study the emergence, stability and evolution of solitons and compactons in a class of Klein-Gordon equations

$$u_{tt} - u_{xx} + u = u^{1+n} - \kappa_{1+2n} u^{1+2n}, \quad n = 1, 2..,$$

endowed with trivial and non-trivial stable equilibria, and demonstrate that similarly to the classical $\kappa_{1+2n} = 0$ cases, solitons are linearly unstable, but their instability weakens as $\kappa_{1+2n} \uparrow$, and vanishes at a critical $\kappa_{1+2n}^{crit} = \frac{1+n}{(2+n)^2}$, where solitons disappear and kinks form.

As the growing amplitude of the unstable soliton approaches the non-trivial equilibrium, it morphs into "meson", a robust box shaped sharp pulse with a flat-top plateau which expands at a sonic speed. In the κ_{1+2n}^{crit} vicinity, where the instability is suppressed, and the internal modes hardly changed, solitons persist for a very long time and rather than turn into meson, convert to breather.

Linear damping tempers the conversion and slows it. When $-1/2 < n < 0$, compactons emerge and being unstable morph either to meson or to breather.

Joint work with Slava Krylov